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B.Sc part-II (Sub), Indeterminate forms. 22-05-2020.
(Diff. Calculus)

Q.1. Evaluate $\lim_{x\to 0} \frac{e^x - e^{-x}}{x}$

Soln: The given limit $= \lim_{x\to 0} \frac{e^x - e^{-x}}{x}$ [form $\frac{0}{0}$]

$= \lim_{x\to 0} \frac{e^x + e^{-x}}{1}$, By L'Hospital's rule

$= 1+1 = 2.$

Q.2. Evaluate $\underset{x\to 0}{Lt} \frac{a^x - 1}{x}$

Soln: It is in the form $\frac{0}{0}$.

$\therefore \underset{x\to 0}{Lt} \frac{a^x-1}{x} = \underset{x\to 0}{Lt} \frac{\frac{d}{dx}(a^x-1)}{\frac{d}{dx}(x)} = \underset{x\to 0}{Lt} \frac{a^x \cdot \log_e a}{1} = \log_e a.$

Q.3 Evaluate $\lim_{x\to 0} \frac{(e^x-1)\tan^2 x}{x^3}$

Soln: It is in the form $\frac{0}{0}$

$\therefore \underset{x\to 0}{Lt} \frac{(e^x-1)\tan^2 x}{x^3} = \lim_{x\to 0} \frac{e^x-1}{x}\left(\frac{\tan x}{x}\right)^2$

$= \lim_{x\to 0} \frac{e^x-1}{x} \cdot \left(\lim_{x\to 0} \frac{\tan x}{x}\right)^2$

$= \lim_{x\to 0} \frac{e^x}{1} \cdot 1^2$ [By L'Hospital rule]

$= 1.$ Ans

Q.4. Find $\lim_{x\to 0} \frac{e^x + e^{-x} - 2}{x^2}$

Soln: It is in the form $\frac{0}{0}$.

$\therefore \lim_{x\to 0} \frac{\frac{d}{dx}(e^x+e^{-x}-2)}{\frac{d}{dx}(x^2)} = \lim_{x\to 0} \frac{e^x - e^{-x}}{2x} = \lim_{x\to 0} \frac{\frac{d}{dx}(e^x - e^{-x})}{\frac{d}{dx}(2x)}$

$= \lim_{x\to 0} \frac{e^x + e^{-x}}{2} = \frac{1+1}{2} = \frac{2}{2} = 1.$

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Q.5 Evaluate $\lim\limits_{x\to 0} \dfrac{xe^x - \log(1+x)}{x^2}$

Soln: It is in the form $\frac{0}{0}$

$$\therefore \lim_{x\to 0} \frac{xe^x - \log(1+x)}{x^2} = \cancel{\lim_{x\to 0} \frac{e^x + xe^x - \frac{1}{1+x}}{2x}}$$

$$= \lim_{x\to 0} \frac{\frac{d}{dx}(xe^x) - \frac{d}{dx}\log(1+x)}{\frac{d}{dx}(x^2)} = \lim_{x\to 0} \frac{e^x + xe^x - \frac{1}{1+x}}{2x}$$

$$\cancel{= \lim_{x\to 0} \frac{e^x + e^x + xe^x + \frac{1}{(1+x)^2}}{2}}$$

$$= \lim_{x\to 0} \frac{\frac{d}{dx}(e^x) + \frac{d}{dx}(xe^x) - \frac{d}{dx}\left(\frac{1}{1+x}\right)}{2\frac{d}{dx}(x)}$$

$$= \lim_{x\to 0} \frac{e^x + e^x + xe^x + \frac{1}{(1+x)^2}}{2} = \frac{3}{2} \text{ Ans}$$

Q.6. Evaluate $\lim\limits_{x\to 0} \dfrac{x\cos x - \log(1+x)}{x^2}$

Soln: It is in the form $\frac{0}{0}$.

$$\therefore \lim_{x\to 0} \frac{x\cos x - \log(1+x)}{x^2} = \lim_{x\to 0} \frac{\frac{d}{dx}(x\cos x) - \frac{d}{dx}\log(1+x)}{\frac{d}{dx}(x^2)}$$

$$= \lim_{x\to 0} \frac{\cos x - x\sin x - \frac{1}{1+x}}{2x}$$

$$= \lim_{x\to 0} \frac{\frac{d}{dx}(\cos x) - \frac{d}{dx}(x\sin x) - \frac{d}{dx}\left(\frac{1}{1+x}\right)}{2\frac{d}{dx}(x)}$$

$$= \lim_{x\to 0} \frac{-\sin x - \sin x - x\cos x + \frac{1}{(1+x)^2}}{2\cdot 1} = \frac{1}{2}$$

Q.7. Evaluate $\lim\limits_{x\to 0} \dfrac{x - \sin x}{x^3}$

Soln: It is in the form $\frac{0}{0}$

$$\therefore \lim_{x\to 0} \frac{x - \sin x}{x^3} = \lim_{x\to 0} \frac{\frac{d}{dx}(x) - \frac{d}{dx}(\sin x)}{\frac{d}{dx}(x^3)} = \lim_{x\to 0} \frac{1 - \cos x}{3x^2} = \lim_{x\to 0} \frac{\frac{d}{dx}(1-\cos x)}{3\frac{d}{dx}(x^2)}$$

$$= \lim_{x\to 0} \frac{\sin x}{6x} = \lim_{x\to 0} \frac{\frac{d}{dx}(\sin x)}{6\frac{d}{dx}(x)} = \lim_{x\to 0} \frac{\cos x}{6} = \frac{1}{6} \text{ Ans}$$

[by L'Hospital's rule]

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